Engg. Mechanics

- Mechanics of rigid bodies
  - Statics
  - Dynamics
    - Kinematics
    - Kinetics
- Mechanics of deformable bodies
  1. Strength of Material
  2. Theory of elasticity
  3. Theory of plasticity
- Mechanics of fluid
  1. Ideal Fluid
  2. Viscous Fluid
  3. Incompressible Fluid

ENGG. MECHANICS

Study of motion of rigid bodies under the action of forces.

Static (rest)
- Equilibrium
- Planar Truss
- Virtual Work
- Centroid
- Moment of Inertia

Dynamics (motion)

1. Kinematics (motion only)
   $\begin{Bmatrix} \vec{s}, \vec{v}, \vec{a} \\ \theta, \omega, \alpha \end{Bmatrix}$
2. Kinetics (motion along with its cause)
   $\begin{Bmatrix} \vec{s}, \vec{v}, \vec{a} \ \& \ \vec{F} \\ \theta, \omega, \alpha \ \& \ T \end{Bmatrix}$

⇒ Study of motion means the "study of parameters which guides the motion."

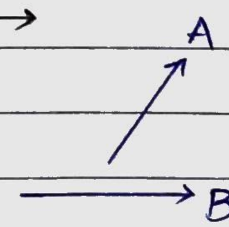
Dynamics (motion)
- Translation
- Friction
- Circular Motion
- Collision / Impact
- Rotation
- General Motion
- Work - Energy Theorem

1. Statics → when the body is at rest
2. Dynamic → when the body is in motion
   (i) Kinematics → when the forces which cause the motion is not considered
   (ii) Kinetics → when the forces behind the motion is considered
3. Scalar Quantity → those physical quantities which only has magnitude
   E.g. → mass, length, time & temp.
4. Vector Quantity → those physical quantities which have magnitude as well as direction.

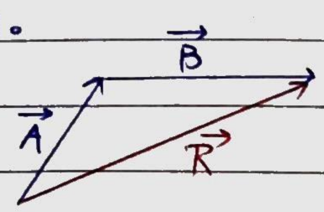
Addition of Vectors →

1. Triangle law of vector addition
2. Polygon law
3. Parallelogram law

1. Triangle law of vector addition →



↳ if $\vec{A}$ & $\vec{B}$ are two vectors
then $\vec{R} = \vec{A} + \vec{B}$
where $\vec{R}$ is the resultant force.
↳ In order addition of vector

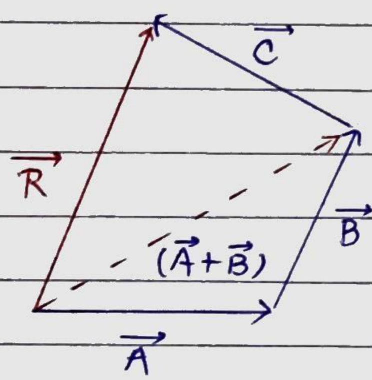


Head to Tail /
Tail to Head Method

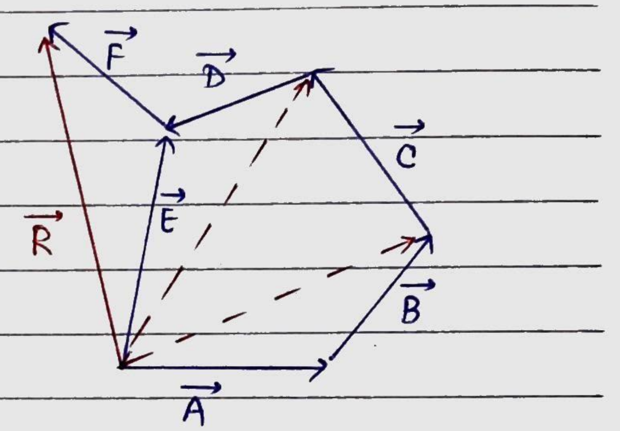
2. Polygon Law of Vector Addition →

↳ this is the extended form of triangle law.

(i) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ (ii) $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F}$

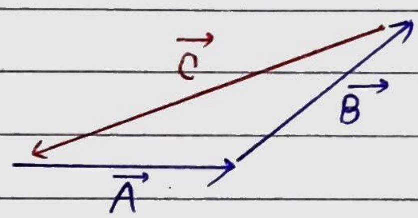


Head to Tail Method



Example → $\vec{A} + \vec{B} + \vec{C} = 0$

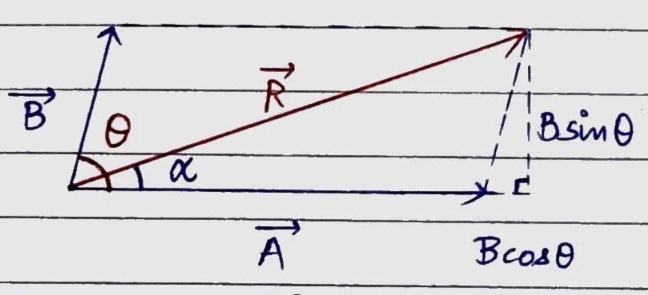
in order addition of vector $\vec{C}$.



$\vec{C} = -(\vec{A} + \vec{B})$

3. Parallelogram Law of Vector Addition →

↳ if $\vec{A}$ & $\vec{B}$ are two vectors
then $\vec{R} = \vec{A} + \vec{B}$

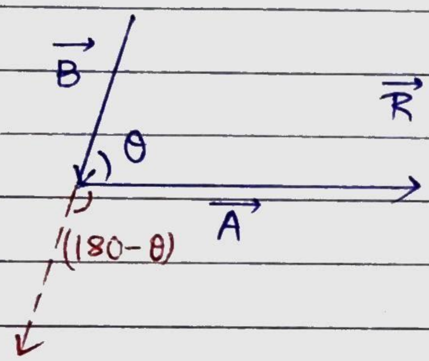


Head to Head Method
OR
Tail to Tail Method

$$R^2 = (A + B\cos\theta)^2 + B^2\sin^2\theta$$
$$= A^2 + B^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}} \rightarrow \text{Magnitude}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)} \longrightarrow \text{Direction}$$



$\vec{R} = \vec{A} - \vec{B}$

Product of Vectors →

1. Dot Product (Scalar Product) →

$$\vec{A}\cdot\vec{B} = \vec{C} = A.B\cos\theta$$

if $\theta = 90^\circ \Rightarrow 0$, $\theta = 0^\circ \Rightarrow +AB$, $\theta = 180^\circ \Rightarrow -AB$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_Y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = A_x\cdot B_x + B_Y\cdot A_Y + A_z\cdot B_z$$

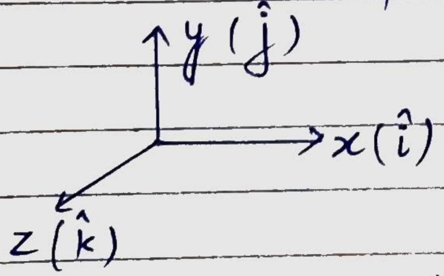
2. Cross Product (Vector Product) →

$$\vec{A}\times\vec{B} = \vec{C} \qquad \vec{B}\times\vec{A} = \vec{D}$$

$A.B\sin\theta\,(\hat{c}) \rightarrow$ unit vector $\qquad |\vec{C}| = AB\sin\theta$

$\hat{c} \rightarrow$ unit vector / direction of vector $\vec{c}$

Direcⁿ → {Right Hand Thumb Rule}



$$\vec{D} = \vec{B}\times\vec{A}$$

$$|\vec{D}| = BA\sin\theta = |\vec{C}|$$

Right hand thumb Rule

$\hat{C}$ = perpendicular to the plane, outward

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

NOTE :- Direction of $\vec{C}$ will be $\perp$ to the plane containing $\vec{A}$ and $\vec{B}$ and will be decided by Right hand thumb rule.

Ques. The resultant of two vectors $\vec{A}$ & $\vec{B}$ is R. If $\vec{B}$ is doubled, then the new resultant is $\perp$ to $\vec{A}$, then :-

(a) A = B (b) A = R (c) B = R (d) None

Solⁿ. $\underbrace{\vec{A} \quad \& \quad \vec{B}}_{\theta} \longrightarrow R$

$$A^2 + B^2 + 2AB\cos\theta = R^2 \quad \text{——} \quad (1)$$

$\underbrace{\vec{A} \quad \& \quad 2\vec{B}}_{\theta} \longrightarrow R'$ is $\perp$ to $\vec{A}$

$$A^2 + 4B^2 + 4AB\cos\theta = (R')^2 \quad \text{——} \quad (2)$$

from 2 and 3 $\rightarrow$

$$A^2 + \cancel{4B^2} + 4AB\cos\theta = \cancel{4B^2} - A^2$$

$$4AB\cos\theta = -2A^2$$

$$2AB\cos\theta = -A^2 \quad \text{——} \quad 4$$

Using value of eq^n. 4 in eq^n. 1

$$A^2 + B^2 - A^2 = R^2$$

$$\boxed{B = R}$$

Ques. The resultant of 2 vectors when acting at right angles is 10 kN. If they act as 60° their resultant is $5\sqrt{6}$ kN. The magnitude of individual vectors are ?

Sol^n. A & B ; $\theta = 90^\circ$; R = 10 kN

$$A^2 + B^2 = 10^2 = 100 \quad \text{——} \quad 1$$

A & B ; $\theta = 60^\circ$; $R = 5\sqrt{6}$ kN

$$A^2 + B^2 + AB = (5\sqrt{6})^2 = 150 \quad \text{——} \quad 2$$

from 1 and 2 –

$$AB = 50$$

$$(A+B)^2 = A^2 + B^2 + 2AB$$
$$= 100 + 2(50)$$
$$= 200$$
$$A + B = \sqrt{200} = 10\sqrt{2}$$

$$2A = 10\sqrt{2}$$

$$(A-B)^2 = A^2 + B^2 - 2AB$$
$$= 100 - 100$$
$$= 0$$
$$A - B = 0$$
$$A = B$$

$$\boxed{A = 5\sqrt{2}\ kN = B}$$

Ques. Two forces are acting at a point making an angle of 120° with each other. If the resultant force is ⊥ to the smaller force which 50 N, then find the larger force.

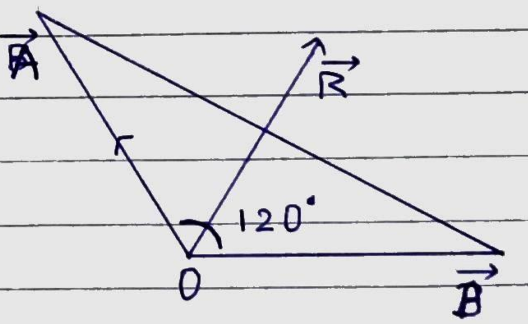
Sol^n: suppose $\vec{B}$ is the smaller force.

$\vec{A} = 50\ N$

$\angle AOB = 120°$

$\angle ROA = 90°$

find $\vec{B} = ?$



Let $\alpha = \angle ROB = 30°$

$$\tan\alpha = \frac{A\sin\theta}{A\cos\theta + B} = \frac{A\sin 120°}{A\cos 120° + B} = \tan 30°$$

$$\Rightarrow (A\cos 120° + B) \times \tan 30° = A\sin 120°$$

$$\Rightarrow -0.288\,A + B = 0.866\,A$$

$$\Rightarrow 1.154\,A = B$$

$$\Rightarrow B = 57.7\ N$$

FORCE SYSTEM

⇒ **Force** :- It is an agent which produce or tends to produce a change in the state of motion of any body.

↳ Characteristics of a force :-

1. Magnitude of the force
2. Line of action of the force
3. Nature of force (push / pull)
4. Point of application

↳ Principle of Transmissibility of forces :-

(pulling or pushing produces the same effect)

If a force acts at any point on a rigid body it may also be considered to act on any other point on its line of action, provided this point is rigidly connected with the body.

↳ System of Forces :-

When two or more forces act on a body, they are called to a system of forces.

1. Coplanar forces → Those forces, whose line of action lie on the same plane.
2. Collinear forces → Those forces, whose line of action lie on the same line.
3. Concurrent forces → Those forces, which meet at one point. These forces may or may not be collinear.
4. Coplanar concurrent forces → The forces, which meet a one point and their line of action also lie on same plane.

5. Coplanar non-concurrent forces :- The forces, which do not meet but their line of action lie on the same plane.

6. Non-coplanar concurrent forces → The forces, which meet at one point but their lines of action do not lie on the same plane.

7. Non-coplanar non-concurrent forces → The forces, which do not meet and their line of action do not lie on same plane.

↳ Composition of Forces :- The process of finding out the resultant forces, of a number of given forces is called composition of forces.

*. The resultant in such a case can be determined by using parallelogram law of vector addition.

↳ Resolution of Forces :- The process of finding two components of a force, without changing its effect on the body is called resolution of forces.

↳ Principle of Resolution :- The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in same direction.

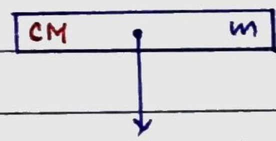
☞ Note → The forces are resolved in vertical and horizontal direction.

⇒ **Polygon Law of Forces** :- If a numbers of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon, their resultant may be represented in magnitude & direction by the closing side of the polygon, taken in opposite order.

⇒ To define a force → Magnitude
→ Direction
→ Point of Application

⇒ **Introduction to frequently appearing force** :–

1. **Weight (Gravity)** :–
due to self mass,
two bodies attract each other



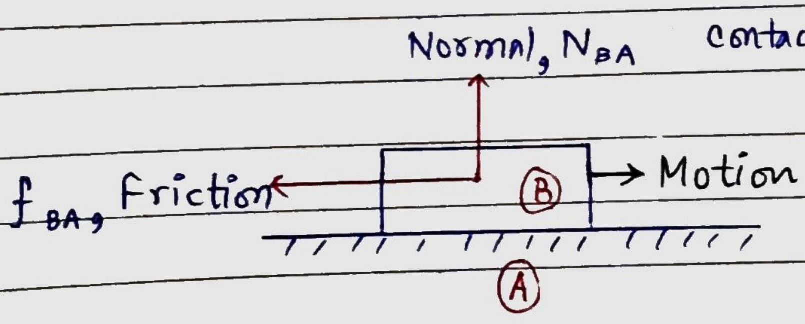
mg = gravity
{on mass 'm' by Earth}

2. Contact Force :-
When two bodies come in contact with each other physically.

{Two component of this force

(i) Normal rxn. (perpendicular to contacting surface)

(ii) Friction (parallel to contacting surface) (when body in motion)

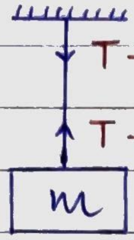


NOTE :- N_{BA} & f_{BA} has no effect on each other.

3. Tension :- When a string is in pulling condition then the force exerted by string on the another body is known as 'Tension'.

Example →

1.



T → on support by string

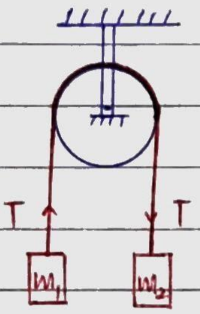
T → on mass 'm' by string

rest → m

Assumption → string need to be massless

2. Pulley →

$$\frac{T_1}{T_2} = e^{\mu\theta}$$



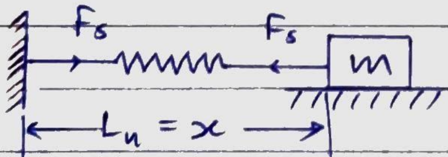
if $m_1 \neq m_2$
then $T_1 \neq T_2$

Tension in string will be same throughout its length if :-

* string is massless
* pulley should be frictionless

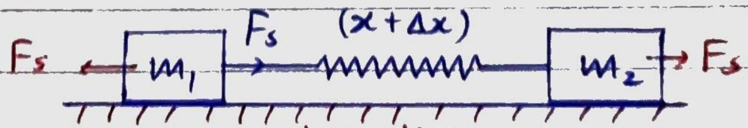
4. Spring force , $F_s = k(\Delta x)$

Δx ↳ elongation or compression from the natural length



{Natural length of spring}

* elongation → pulling
* compression → pushing



elongation

Spring	string	Bar
push ✓	pull ✓	pull ✓
pull ✓	push ✗	push ✓

⇒ Newton's First Law (NFL) ⇒ used to find unknown force
{Law of Inertia}

Inertial Frame

For a particle,
if net force on a particle is zero, $\Sigma \vec{F} = 0$
then the particle has no acceleration, $\vec{a} = 0$

$\vec{a} = 0$

rest

uniform linear velocity
(constant velocity)

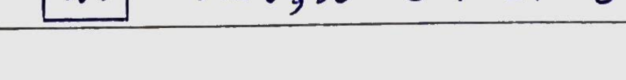
Inertia
↳ unwillingness to change

For a rigid body,
if a net external force on the body is zero, $\Sigma \vec{F}_{ext} = 0$ then the acceleration of centre of mass will be zero, $\vec{a}_{CM} = 0$.

Application of NFL →

① $T - mg = 0$ → NFL

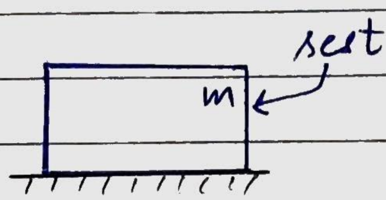
$T = mg$ unknown found



② $\vec{a} = 0$

$N - mg = 0$

$N = mg$



⇒ Newton's Third Law of Motion →

if there is an action on one body, then there must be a reaction on another body which is equal & opposite.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

*. NTL is applicable only when there are two bodies.

Direction	Nature
opposite	same

⇒ Equilibrium ⇒ two condition for rigid body

1. $\Sigma \vec{F} = 0$ Net force = zero

$\Sigma F_x = 0 = \Sigma F_y = \Sigma F_z = 0$

2. $\Sigma \vec{\tau} = 0$ Net torque = 0

about any pt. or line in space

{Equilibrium}

⇒ Rest

⇒ Uniform linear velocity

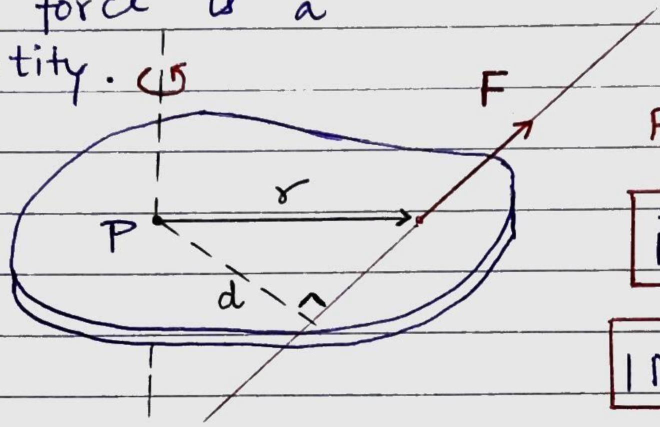
→ For particle → $\Sigma \vec{F} = 0$

→ For rigid body → $\Sigma \vec{F} = 0$ & $\Sigma \vec{\tau} = 0$

⇒ Moment of a Force :- It is the measure of tendency of the force to rotate the body about the point of interest P.

*. Moment of force is a vector quantity.

*. $\vec{r}$ is the position vector.



Formulas →

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

$$\boxed{|M| = Fd}$$

*. It is the perpendicular distance from the pivot point to the line of action of the force.

*. While defining the moment of a force not only magnitude but also the sense of rotation needs to be specified.

*. Right-hand thumb rule for direction

*. Units -
kN-m ; N-mm

↳ Types of Moments :- 1. Clockwise 2. Anti-clockwise

☞ NOTE → General convention :-
clockwise moment is positive
anticlockwise moment is negative.

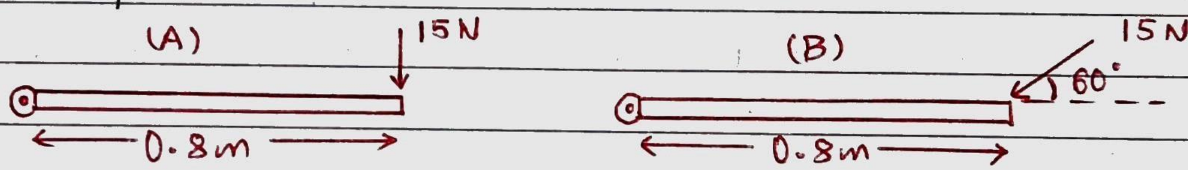
↳ Varignon's Principle of Moments →

If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

$$\vec{R} \times \vec{r} = \vec{F_1} \times \vec{r_1} + \vec{F_2} \times \vec{r_2} + \dots + \vec{F_n} \times \vec{r_n}$$

Ques. A force of 15N is applied ⊥ to the edge of a door 0.8 m wide. Find the moment of force about hinge.
If the force is applied at an angle of 60° to the edge of the same door, find the moment of this force.

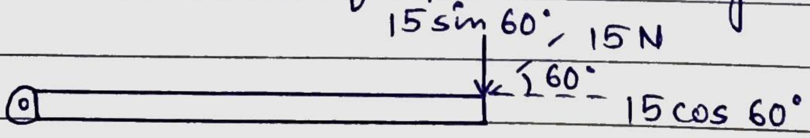
Sol^n.



Given : force, P = 15 N
width of door, l = 0.8 m

(A) Moment when force is ⊥ to the edge
= $P \times l$ = 15×0.8 = 12 Nm

(B) Resolving the component of force on edge -



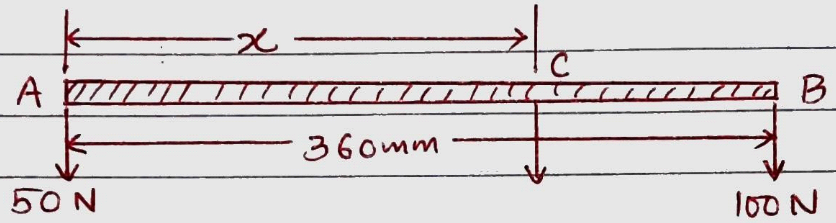
Now, the force should be : $P' = 15 \sin 60°$
= 13 N

∴ Moment = 13×0.8 = 10.4 Nm.

⇒ **Parallel Forces** :- Those force which have parallel line of action.

1. Like parallel force
2. Unlike parallel force

Ques. Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.



Solⁿ: Magnitude of resultant force, $R = 50 + 100 = 150\,N$

Point where resultant acts : Let x

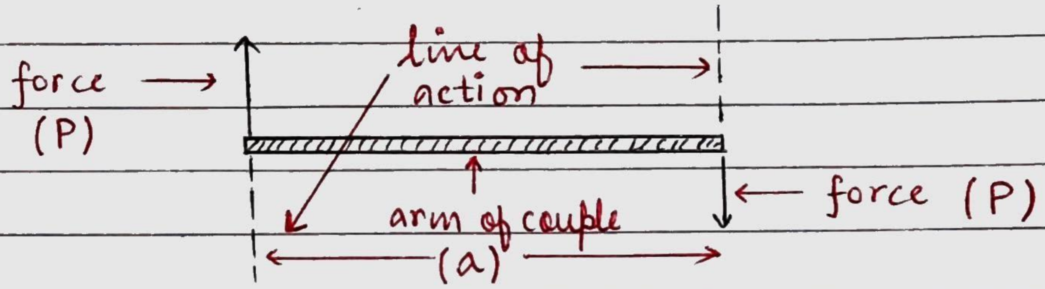
Now, taking clockwise & anticlockwise moments of the forces about C and equating –

$$50 \times x = 100 \times (360 - x)$$

$$150x = 36000 - 100x$$

$$x = \frac{36000}{150} = 240\,mm$$

⇒ **Couple** :– A pair of two equal and unlike parallel forces (forces equal in magnitude, with lines of action parallel to each other and acting in opposite direction) is known as a couple.



* The movement of the whole body is not possible because the resultant forces is zero.
* The perpendicular distance b/w the line of action of the two forces is called the arm of couple.

↳ **Moment of a Couple** :– It is the product of the force and the arm of the couple. It is also known as torque.

Moment of a couple = $P \times a$

where P = magnitude of force

a = arm of couple

— EQUILIBRIUM —

⇒ Equilibrium :-

→ Analysis of forces and determination of resultant of forces is common to both statics and dynamics.

→ In statics, the body is said to be in equilibrium when,

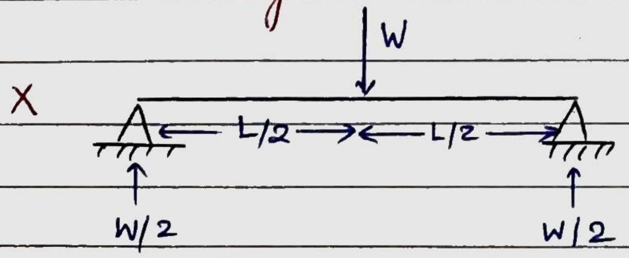
$$\Sigma \vec{F} = 0 \; ; \; \Sigma \vec{M} = 0$$

→ In dynamics, non-zero resultants of force system and their effect is studied on motion of particle, a rigid body or a system of rigid bodies.

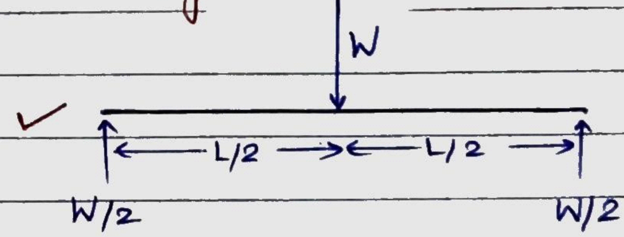
⇒ **Free-Body Diagram** :- It is a diagram of an isolated body or a postion along with the net effects of the force system on it is called a free-body diagram.

→ Free Body Diagram consist of the system and forces from surrounding and surrounding should not be visible.

Wrong FBD shown

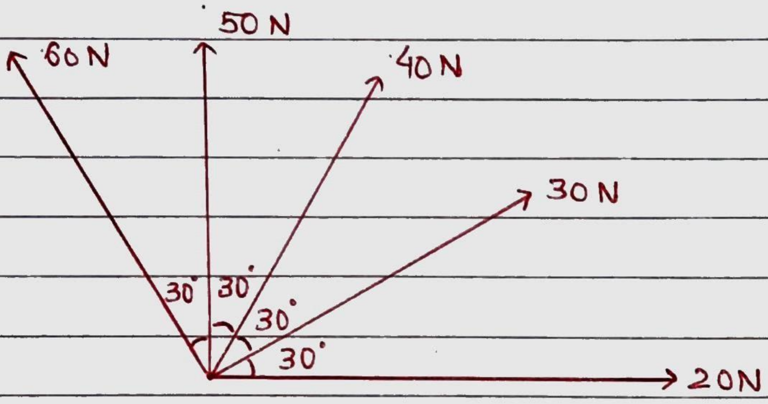


Right FBD shown



Ques. The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, toward the other five angular point. Find the magnitude and direction of the resultant force.

Sol^n:



Magnitude of the resultant force :

Resolving all forces -

(i) $\Sigma F_x = 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°$

$= 20 + 25.98 + 20 + (-30)$

$= 36$ N

(ii) $\Sigma F_Y = 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120°$

$= 15 + 34.64 + 50 + 51.96$

$= 151.6$ N

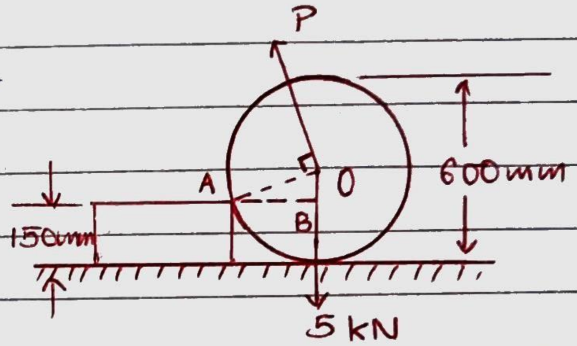
Therefore, resultant $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_Y)^2} = 155.8$ N

Direction of the resultant force :

$\tan\alpha = \dfrac{\Sigma F_Y}{\Sigma F_x} = \dfrac{151.6}{36} = 4.211 \Rightarrow \alpha = 76.6°$

Ques. A uniform wheel of 600mm diameter, weighing 5kN rests against a rigid rectangular block of 150 mm height.
Find the least pull, through the centre of the wheel, required to turn the wheel over the corner A of the block. Also find the rxn. on the block. Take all the surface smooth.

{Concept → Three force system}



Sol^n: Let P = least pull required just to turn the wheel in kN.

$AB = \sqrt{(300)^2 - (150)^2} = 260mm$

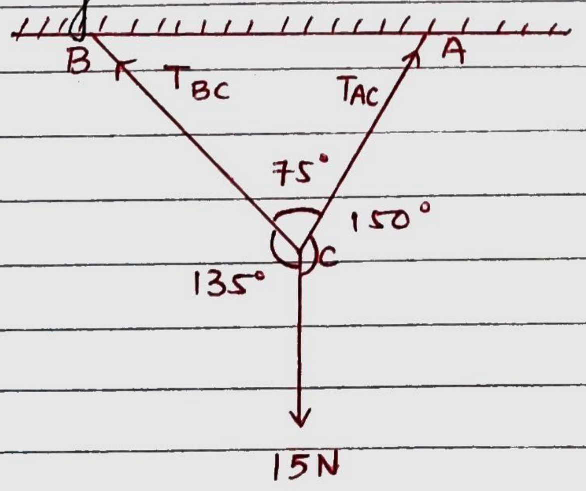
Moment about A → $P \times 300 = 5 \times 260$

$P = 4.33$ kN Ans

Reaction on the block → $R \cos 30^\circ = P \sin 30^\circ$

$\therefore R = \dfrac{4.33 \times 0.5}{0.866} = 2.5$ kN. Ans

Ques. Determine the forces in the string AC and BC in the system shown below :-



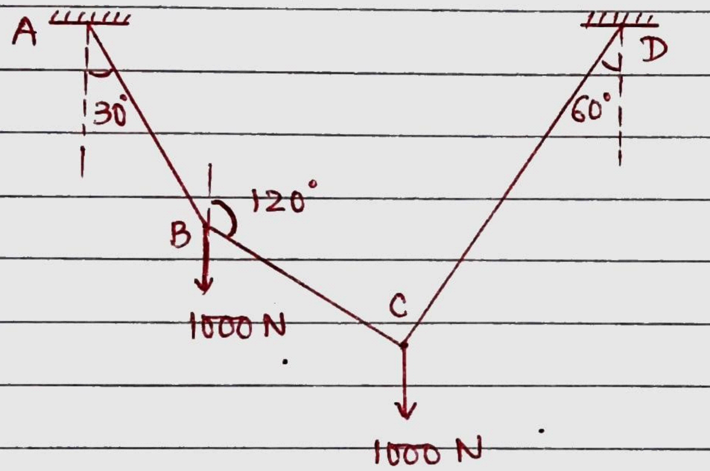
Using Lami's theorem :-

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$\Rightarrow \quad T_{AC} = 10.98 \text{ N}$

$\Rightarrow \quad T_{BC} = 7.76 \text{ N}$

Ques. Determine the tension in the portion AB, BC and CD of the string shown below.



$\left.\begin{matrix} T_{AB} \\ T_{BC} \\ T_{CD} \end{matrix}\right\}$ find

Sol^n: Let T_{AB} = tension in AB

T_{BC} = tension in BC

T_{CD} = tension in CD

Using Lami's theorem :- (i) At joint B

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

$T_{AB} = 1732 \text{ N}$ $\quad T_{BC} = 1000 \text{ N}$

(ii) At joint C

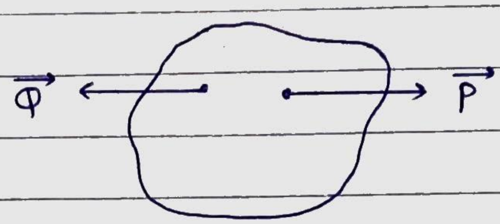
$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$\therefore \quad T_{CD} = 1000 \text{ N}$

⇒ Principle of Equilibrium :-

1. Two force principle → if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

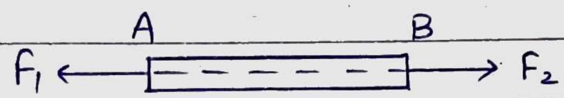
1. $\vec{P} + \vec{Q} = 0$



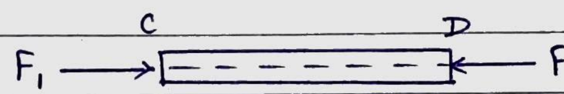
$\vec{P} = -\vec{Q}$

2. $\Sigma \vec{M} = 0$
↳ collinear

Application :- Truss Member

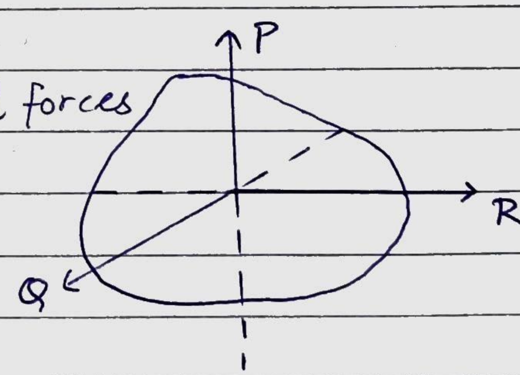


$F_{AB} = F_1$ (tensile)



$F_{CD} = F_2$ (compressive)

2. Three force principle →
$\vec{P}$, $\vec{Q}$ & $\vec{R}$ are non-parallel forces



1. $\underbrace{\vec{P} + \vec{Q} + \vec{R}}_{\text{coplanar forces}} = 0$

2. $\Sigma \vec{M} = 0$
↳ concurrent

To keep the system in equilibrium they must be coplanar and concurrent.

→ Types of Equilibrium :-

1. Stable Equilibrium → the body returns back to its original position, after it is slightly displaced from its position of rest.

total potential energy, V = minimum

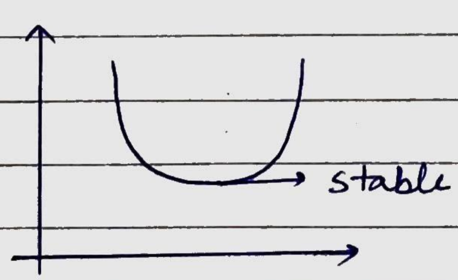
2. Unstable Equilibrium → the body does not returns back to its original position after being displaced from rest.

total potential energy, V = maximum

3. Neutral Equilibrium → the body occupies a new position after slightly displaced from first position of rest.
(height of centre of mass remain at same height)
total potential energy, V = constant

Case I :- Single DOF system →
no. of independent = 1 → (say x)
V = f(x)

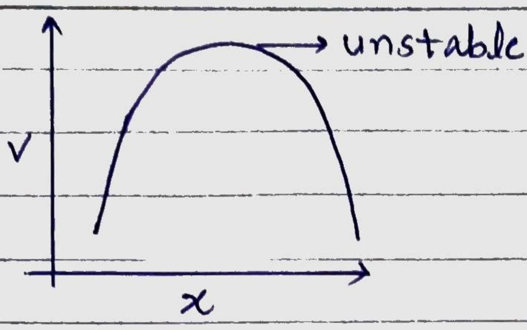
(i) Stable



$$\frac{dV}{dx} = 0$$

$$\frac{d^2V}{dx^2} > 0$$

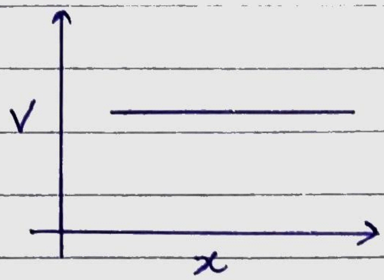
(ii) Unstable



$$\frac{dV}{dx} = 0$$

$$\frac{d^2V}{dx^2} < 0$$

(iii) Neutral



$$\frac{dV}{dx} = \frac{d^2V}{dx^2} = \frac{d^3V}{dx^3} = 0$$

Case 2 : If multiple DOF system

No. of independent variable > 1 (x, y, z)

$$V = f(x, y, z)$$

⇒ For stable, unstable & Neutral eq^n^.

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} \text{ ----- } = 0$$